Motivation and Mathematics Achievement: A Structural Equation Analysis

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Background

Theories of motivation

Self-Determination Theory (Deci & Ryan, 1985) is a theory of motivation (e.g., *autonomous* vs. *controlled* regulation).

TIMSS (1999) study

Previous studies (Casey, 2001; House, 2000, 2003; McLeod, 1992; Papanastasiou, 2002; Ramserier, 1991; Schreiber, 2002) found that students' attitudes and self beliefs were significantly associated with their math achievement internationally.

Overview

Goals of present study

The major purposes of the present study are as follows:

1. Develop a model of the effects of motivation on mathematics performance and achievement.

2. Examine how autonomy support in the classroom predicts mathematics achievement through the mediator of math self-concept.

3. Examine how math self-concept explains additional and significant variance in mathematics achievement, after we control for the effect of intrinsic motivation on mathematics achievement.

Self-Determination Theory

Motivational needs form a continuum:

Intrinsic Motivation ↑   Extrinsic Motivation ↑   Amotivation ↑

Motivation (Deci & Ryan, 1985; Ryan & Connell, 1989)

1) Intrinsic motivation (enjoyment; fun)
2) Extrinsic motivation
External regulation (rule following; reward seeking; avoidance of punishment)
Introjected regulation (self-and other-approval; avoidance of disapproval)
Identified regulation (self-valued goal; personal importance)
3) Amotivation

Vallerand's Model of Intrinsic & Extrinsic Motivation

Social factors	Psychological factors	Motivation	Consequences
Mastery climate	Perceptions of	Intrinsic Motivation	
Performance climate	- Autonomy	External Regulation	
Origin climate	- Competence	Introjected Regulation	
	- Relatedness	Identified Regulation	
		Amotivation	

Figure 1. Vallerand's Model of Intrinsic and Extrinsic Motivation, Vallerand (1999), *Journal of Applied Sport Psychology, 11*.

Mathematics and Motivation

Previous studies (House, 2002, 2003; Mullis et al., 2000 Papanastasiou, 2002; Schreiber, 2002; Schreiber & Chambers, 2003) found that students' attitudes and self-beliefs were significantly associated with their mathematics achievement.

Ramserier (1991) found that self-determined motivation of Swiss students was positively related to their mathematics achievement.

The Hypothesized Model

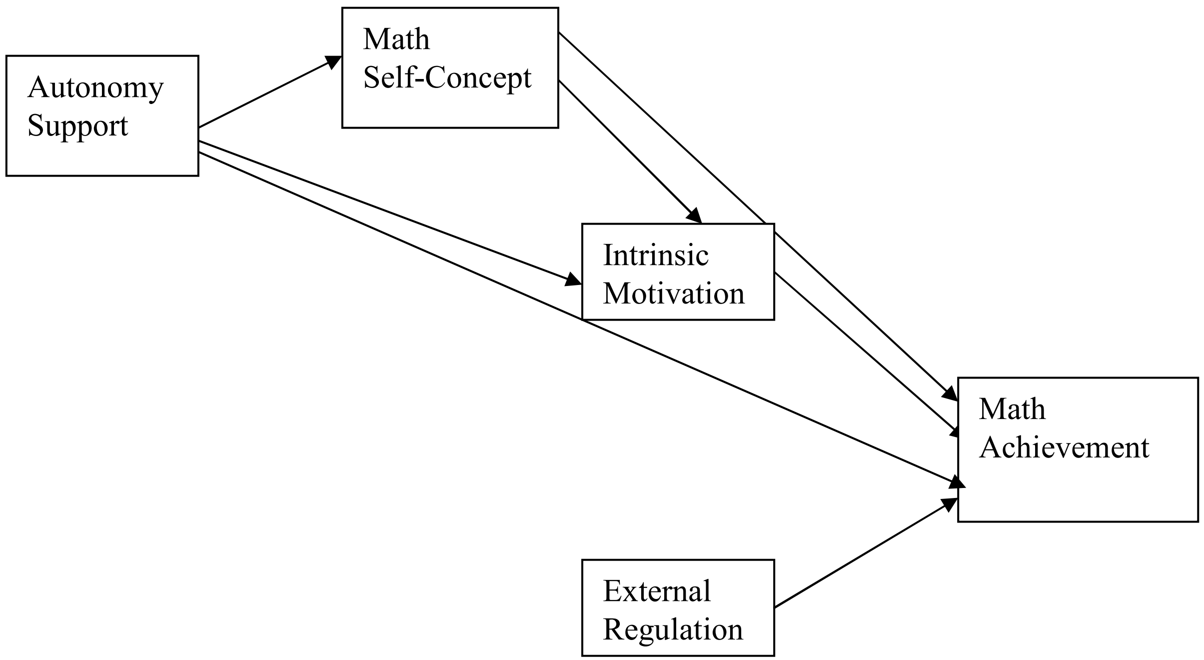

Figure 2. The Motivational Model of Math achievement.

Method: *The TIMSS-R (1999) USA Sample*

Participants in the TIMSS-R (1999) study were 9,072 eighth-grade students, ranging in age from 13 to 14 years, from 150 sampled schools in the U.S..

At the end of the 1998-1999 school year, the subjects took a 90-minute test including 162 math items (Mullis et al., 2000).

In the present study, about 4, 566 students and 449 classrooms were used.

Method: *Measures*

Table 1

List of Items and Scoring

Item		Scoring
AUTONOMY SUPPORT IN THE CLASSROOM		
BTBMLES1	"In math lessons, how often do students work individually without assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES2	"In math lessons, how often do students work individually with assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES4	"In math lessons, how often do students work together as a class with students responding to one another?"	1(Never/almost never) – 4 (every lesson)
BTBMLES5	"In math lessons, how often do students work in pairs or small groups without assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)
BTBMLES6	"In math lessons, how often do students work in pairs or small groups with assistance from the teacher?"	1(Never/almost never) – 4 (every lesson)

Method: *Measures*

Table 1
List of Items and Scoring

Item		Scoring
INTRINSIC MOTIVATION (enjoyment, fun)		
BSBMENJY	"I enjoy learning mathematics."	1(strongly agree) – 4 (strongly disagree)
BSBMLIKM	"I like mathematics."	1 (Like a lot) - 4 (Dislike a lot)
BSBMBORE	"Math is boring." (reversed)	1 (strongly agree) –4 (strongly disagree)
EXTERNAL REGULATION (rule following; reward seeking; avoidance of punishment)		
BSBMJOB	"I need to do well in math to get the job I want."	1(strongly agree) – 4 (strongly disagree)
BSBMWORK	"I would like a job that involved using math."	1(strongly agree) - 4(strongly disagree)
BSBMSCHL	"I need to do well in math to get into the <secondary school> I prefer."	1 (strongly agree) – 4 (strongly disagree)

Method: *Measures*

Table 1

List of Items and Scoring (continued)

Item		Scoring
INTROJECTED REGULATION (self-and other-approval; avoidance of disapproval)		
BSBMPRNT	"I need to do well in mathematics to please my parents."	1 (strongly agree) - 4 (strongly disagree)
BSBMSELF	"I need to do well in mathematics to please myself."	1 (strongly agree) - 4 (strongly disagree)
BSBMMIP2	"My mother thinks that it is important for me to do well in math at school."	1 (strongly agree) - 4 (strongly disagree)
BSBMFIP2	"Most of my friends think it is important for me to do well in math at school."	1 (strongly agree) - 4 (strongly disagree)
BSBMSIP2	"I think it is important to do well in math at school."	1(strongly agree) – 4 (strongly disagree)

Method: *Measures*

Table 1

List of Items and Scoring (continued)

Item		Scoring
MATH SELF CONCEPT (assessment of personal characteristics regarding math ability)		
BSBMMYT1	"I would like mathematics much more if it were not so difficult."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT2	"Although I do my best, mathematics is more difficult for me than for classmates."	1 (strongly agree) - 4 (strong disagree)
BSBMMYT3	"Nobody can be good in every subject, and I am just not talented in mathematics."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT4	"Sometimes when I do not understand a new topic in mathematics initially, I know I will never understand it."	1 (strongly agree) - 4 (strongly disagree)
BSBMMYT5	"Mathematics is not one of my strengths."	1 (strongly agree) - 4 (strongly disagree)

Method: *Measures*

Table 1

List of Items and Scoring (continued)

Item	Scoring
MATH ACHIEVEMENT	
First math overall plausible value	

Note. Plausible value = a measure of total score (but adjusted to make different booklets comparable)

Method: *Reliabilities of subscales*

Most of the alpha coefficients ranged from .71 to .88:

autonomy support - five items (α = .71)
intrinsic motivation - three items (α = .88)
external regulation – three items (α = .73)
introjected regulation – five items (α = .71)
math self-concept - five items (α = .86)

Results: *Correlation Analyses*

Table 2

Correlation Matrix for the Model Variables

	1	2	3	4	5	6
1. Autonomy support	1					
2. Intrinsic motivation	.03	1				
3. External regulation	-.01	-.51	1			
4. Introjected regulation	.03	.36	.52	1		
5. Math self concept	.10	-.56	-.30	-.14	1	
6. Math achievement	.15	.15	-.12	-.17	.45	1

Note. All coefficients above .10 are significant at the .001 level.

Results: *Structural Equation Modeling*

Table 3
Model Evaluation

Model fit	function	fit
Overall Fit		
Chi-square test		
$\chi 2/df$		ratio of 3 or less: good fit
Incremental Fit Indices		
CFI	compare the lack of fit of the hypothesized model with the independence model	values>.95: good fits
NNFI	includes a correction for model complexity	values>.95: good fits
RMSEA	measure the amount of unfitted residuals between the implied and observed covariance matrices	values<.05: close fits
Absolute Fit Index		
SRMR	measure the degree to which the a priori structure reproduces the data	values<.08: close fit
Information Criterion		
AIC	find parsimonious models that best fit the observed data	lowest value
BIC	find parsimonious models that best fit the observed data	

Note. CFI = comparative fit inde; TLI = NNFI = nonnormed fir index; RMSEA = root-mean-square error of approximation; SRMR=standardized root mean-square residual; AIC=akaike information criterion; BIC= bayesian information criterion.

Results: *Structural Equation Modeling*

Table 4

Fit Indices for each of the SEM Models – Autonomy Support treated as Level-1

Model	χ2	df	CFI	TLI	RMSEA	SRMR	AIC	BIC
Model 1	87.183	10	0.951	0.957	0.049	0.043	71416.771	71506.409
Model 2	95.079	11	0.952	0.919	0.058	0.045	71422.667	71506.703
Model 3	92.170	13	0.959	0.951	0.049	0.044	71415.758	71488.189
Model 4	108.457	12	0.927	0.877	0.063	0.045	71424.045	71502.478
Model 5	113.173	14	0.925	0.893	0.059	0.047	71424.760	71491.989
Model 6	116.936	13	0.922	0.867	0.063	0.046	71430.524	71503.355
Model 7	113.616	14	0.925	0.882	0.060	0.047	71425.203	71492.432
Model 8	117.071	15	0.923	0.897	0.058	0.047	71426.659	71488.285

Note. CFI = comparative fit index; TLI = NNFI = nonnormed fit index; RMSEA = root-mean-square error of approximation; SRMR = standardized root mean-square residual; AIC = akaike information criterion; BIC = bayesian information criterion; Model 1 = all free; Model 2 = one path (ME-> SC) was fixed to zero. Model 3=four paths (ME->SC, AS->IM, G->IM, and G-> MP) were fixed to zero; Model 4=two paths (G->IM, ME->SC) were fixed to zero; Model 5=four paths (AS -> SC, AS -> IM, G->IM, ME->SC) were fixed to zero; Model 6 = three paths (G->SC, G->IM, ME->SC) were fixed to zero; Model 7= four paths (AS->IM, SC, MP, and ME->SC) were fixed to zero; Model 8 = five paths (AS->IM, SC, MP, ME->SC, G->IM) were fixed to zero; ME = Mother's education; SC = Math self-concept; AS = Autonomy support; IM = Intrinsic motivation; G = Gender; MP = Math performance.

Results: *Structural Equation Modeling*

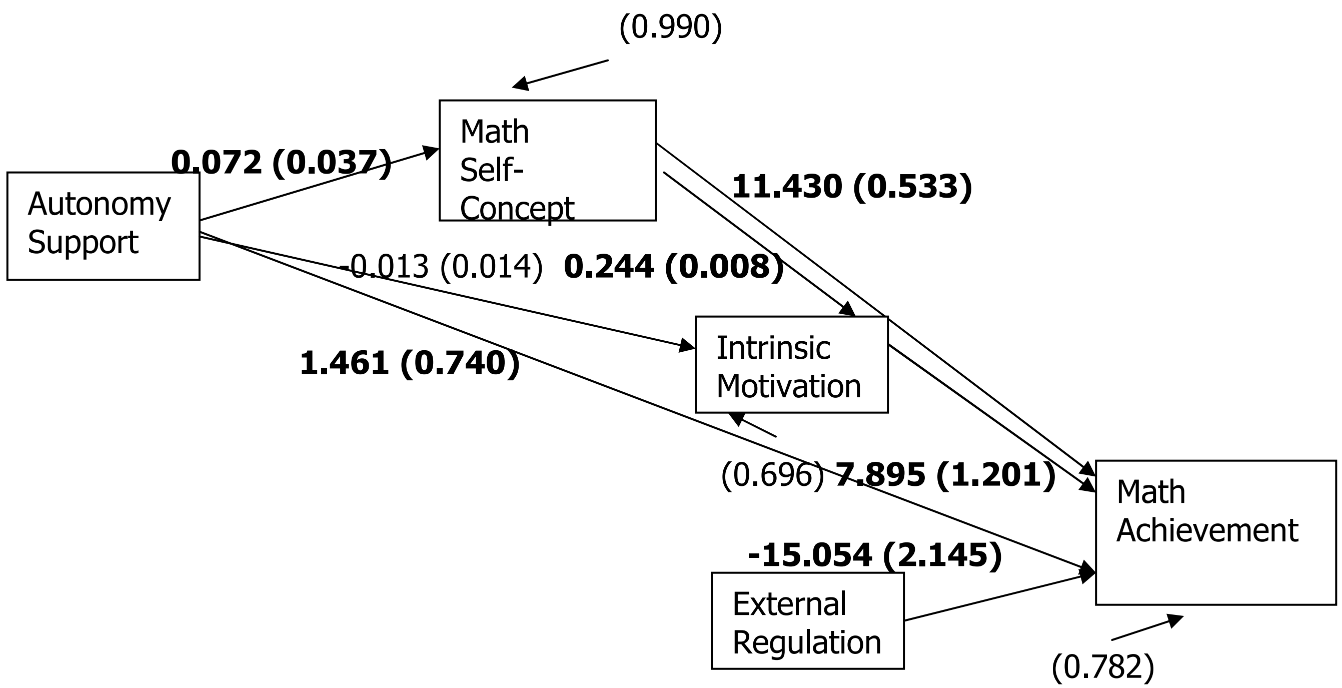

Figure 3. Results for the full model.

Results: *Structural Equation Modeling*

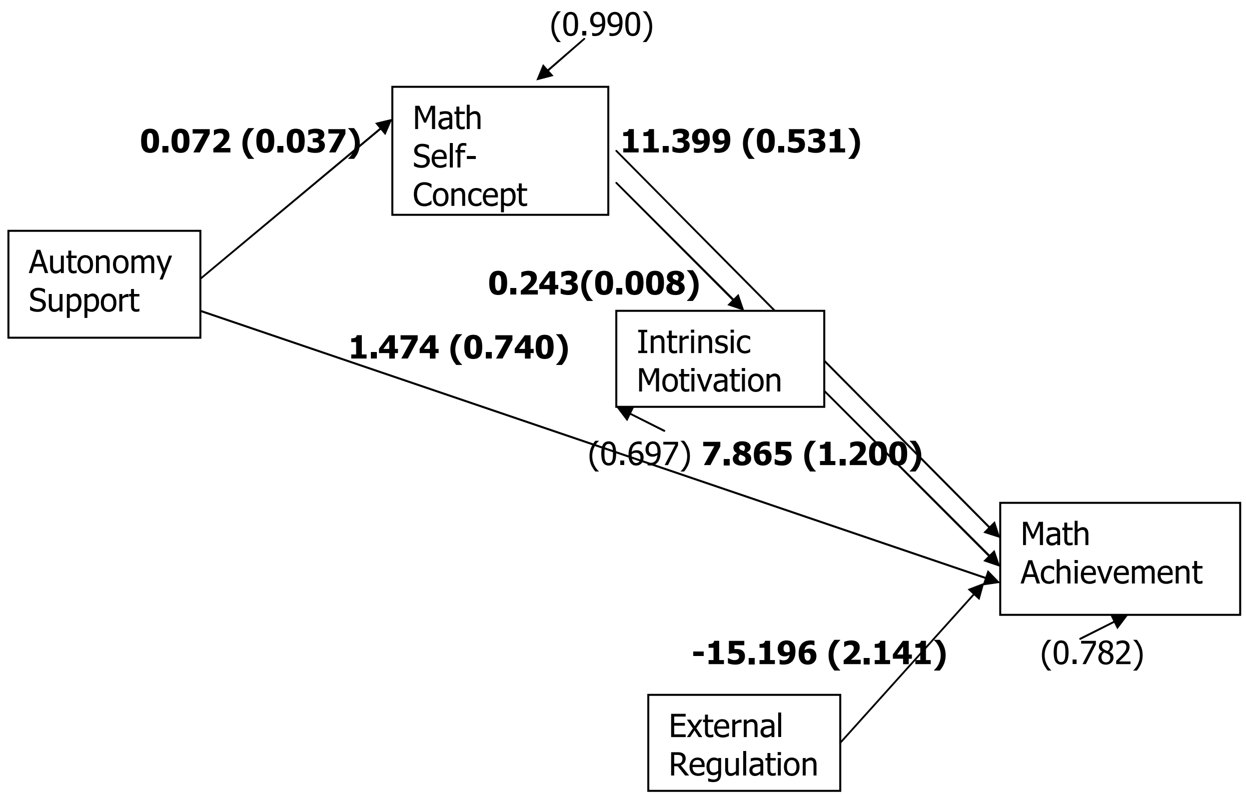

Figure 4. Results for the trimmed full model.

Results: *Multi-level Path Modeling*

We examined the intraclass correlation for math achievement, which was .53.

Results: *Multi-level Path Modeling*

Table 5

Fit Indices for Each of the Multilevel Path Model

Model	χ2	df	CFI	TLI	RMSEA	SRMR (B)	SRMR (W)	AIC	BIC
WL1	134.712	5	0.942	0.825	0.080	0.016	0.039	79996.935	80091.316
WL2	137.095	6	0.941	0.862	0.071	0.016	0.040	80106.526	80106.526
WL3	132.855	7	0.954	0.971	0.049	0.015	0.040	79995.384	80108.641
ML1	279.788	6	0.877	0.734	0.107	0.182	0.045	80085.855	80180.236
ML2	91.665	5	0.959	0.971	0.049	0.001	0.039	79766.210	79917.467
ML3	284.274	6	0.870	0.674	0.108	0.203	0.043	79944.952	80058.209
ML4	78.054	3	0.961	0.901	0.064	0.009	0.038	79799.615	80058.624

Note. WL etc = Within-level Model; ML etc = Multi-level Model; ML Model 1 = Multi-level version of the trimmed full model with NO AS; ML Model 2 = Multi-level version of the trimmed full model; ML Model 3 = Multi-level version of the full model with NO AS; ML Model 4 = Multi-level version of the full model; AS= Autonomy Support.

Results: *Multi-level Path Modeling*

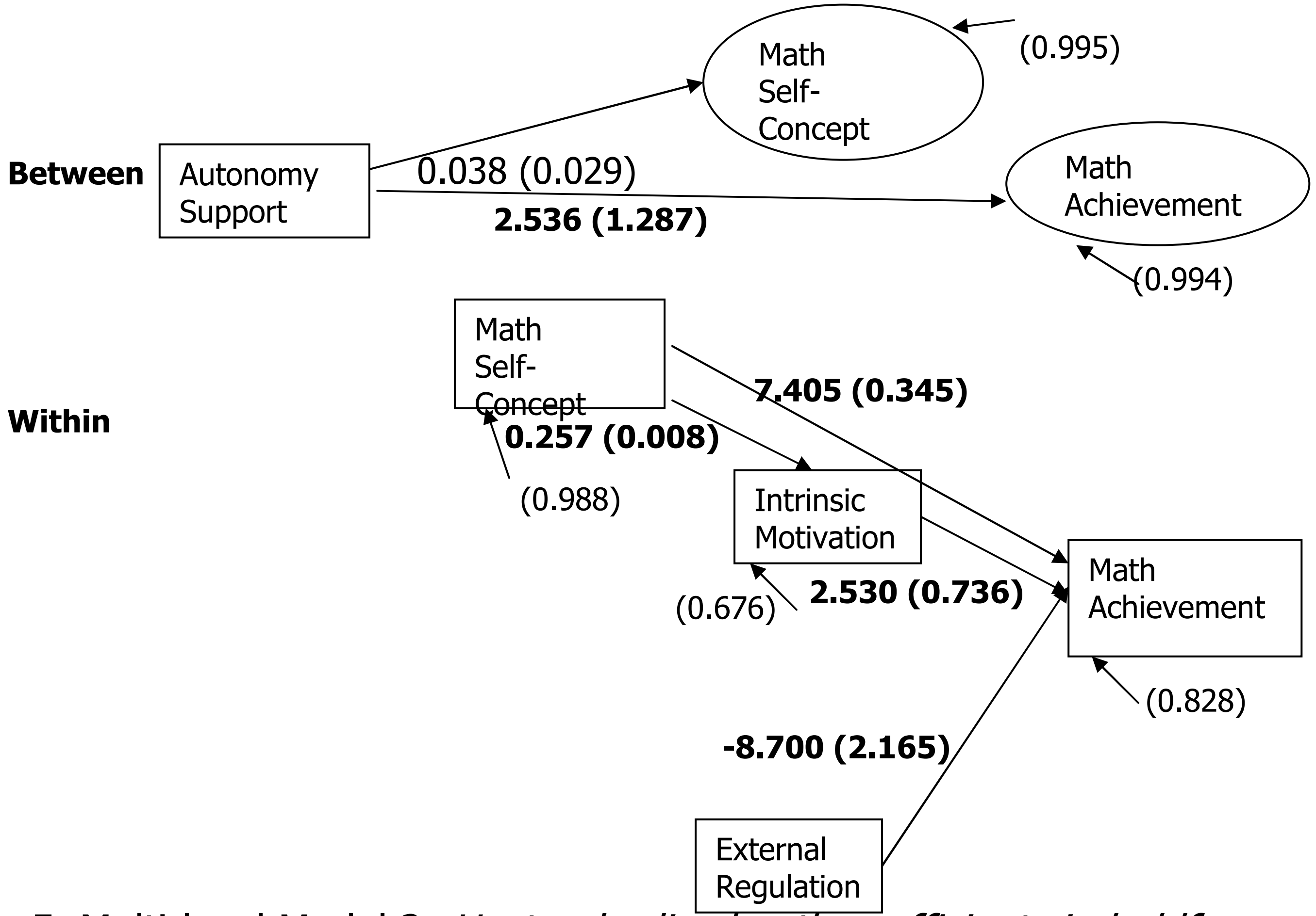

Figure 5. Multi-level Model 2. *Unstandardized path coefficients in boldface are significant.*

Results: *Multi-level Path Modeling*

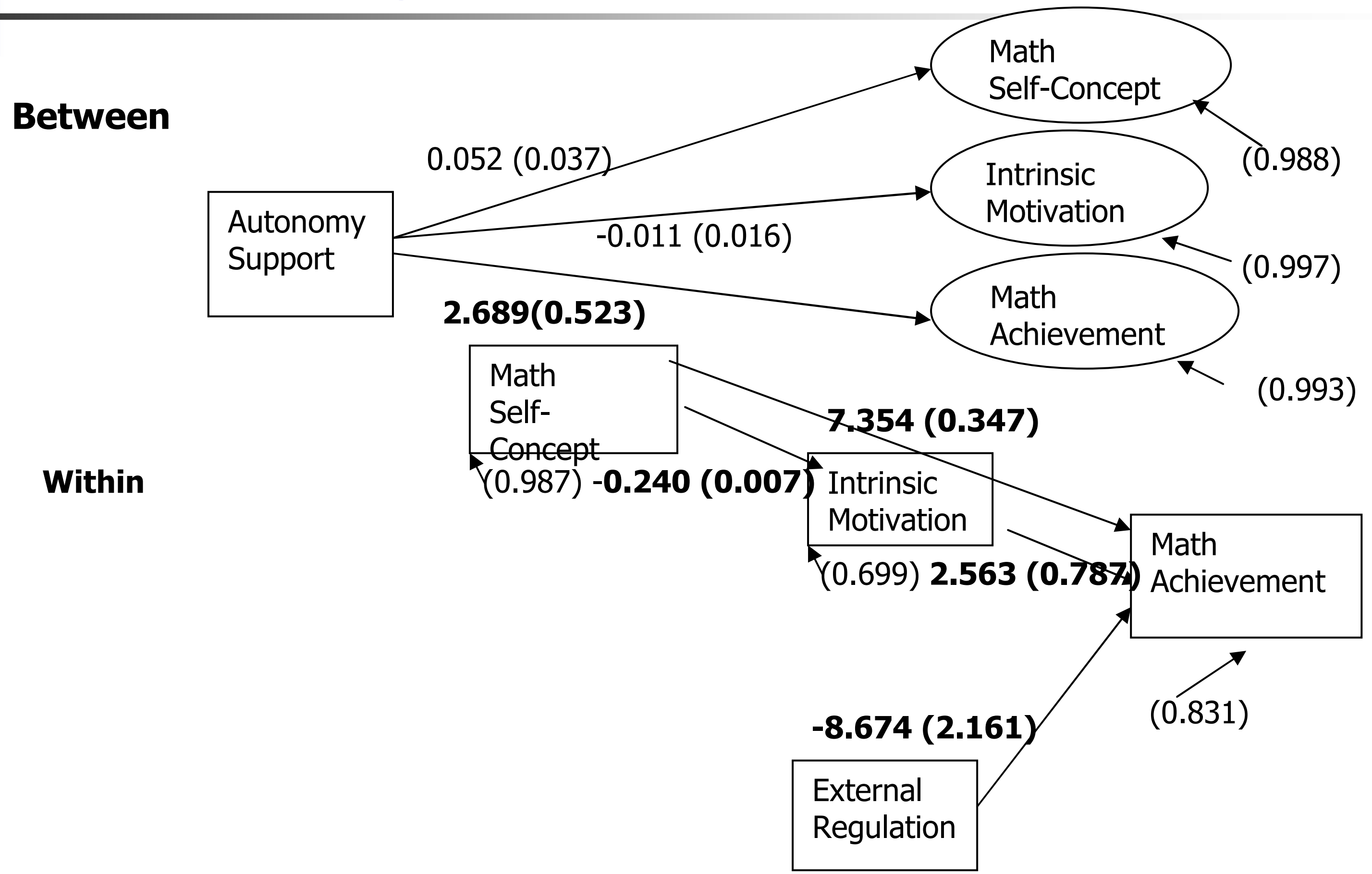

Figure 6. Multi-level Model 4. *Unstandardized path coefficients in boldface are significant.*

Conclusion

Intrinsic motivation positively affected mathematics achievement, whereas external regulation negatively affected math achievement.

Math self-concept explained significant variance in math achievement, controlling for the mediating effect of intrinsic motivation on math achievement.

Autonomy support significantly predicted math achievement.